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On a certain class of connected symmetric trivalent graphs

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ON A CERTAIN CLASS OF CONNECTED SYMMETRIC TRIVALENT GRAPHS

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1. Introduction. A finite group G is called a *T-group* if (1) G is generated by an element a of order two and an element h of order three, and (2) the order of G is larger than six. If a T-group G is given, then a connected symmetric trivalent graph Γ is constructed as follows. The vertices of Γ are the cosets of G with respect to a subgroup $H=\langle h\rangle$. A vertex Hx $(x\in G)$ is joined to Hax, $Hahx$ and Hah^2x. Then clearly G acts as an automorphism group of Γ by right multiplication. The conditions (1) and (2) guarantee that Γ is connected, and that Hax, $Hahx$ and Hah^2x are distinct. So Γ is a connected symmetric trivalent graph. For simplicity we call a graph Γ thus constructed a *T-graph*. For group-theoretical construction of more general connected symmetric trivalent graphs see [5].

Now it seems to the author that T-groups are intrinsically related to non-Abelian simple groups. Namely if S is a non-Abelian simple group, then consider the wreath product $G=\langle h\rangle \wr (S\times S\times S)$ of S with a cyclic group $\langle h\rangle$ of order three. G is a T-group if and only if S satisfies the following condition (!):

(!) S is generated by three elements b, c and d such that there is no automorphism of S permuting b, c and d cyclically.

If S possesses an automorphism ρ permuting b, c and d cyclically, then the holomorph of S with respect to $\langle\rho\rangle$,$\langle\rho\rangle S$, is a T-group. For a proof see §2.

The purpose of this note is (i) to prove some basic propositions on graphs Γ constructed from the wreath product G of the above type and (ii) to determine the automorphism group of Γ. Here the author would like to mention that wide classes of non-Abelian simple groups satisfy (!), though not every non-Abelian simple group satisfies (!) ([6], [9]). Some of our arguments are based on the classification of simple groups of finite orders.

Notation. Let X be a finite group. $|X|$ denotes the order of X. Let $x_1, x_2, \cdots, x_k$ be elements of X. $\langle x_1, x_2, \cdots, x_k\rangle$ is a subgroup of X generated by $x_1, x_2, \cdots, x_k$. For $x\in X$ $|x|=|\langle x\rangle|$.

2. Basic propositions.

Proposition 1. *Let S be a non-Abelian simple group and G the wreath product of S with respect to a cyclic group $H=\langle h\rangle$ of order three. Then G is a T-group if and only if S satisfies (!). If S is generated by three elements b, c and d of order two and if there exists an automorphism h of S permuting b, c and d cyclically, then the holomorph G^* of G with respect to a cyclic group $\langle h\rangle$ is a T-group.*

Proof. We suffix 1, 2 and 3 to $S\times S\times S$ so that the mapping $x\rho_i=x_i$ ($x\in S$, $x_i\in S_i$) is an isomorphism between S and S_i ($i=1, 2, 3$). Then the action of h on $S_1\times S_2\times S_3$ can be described as $h^{-1}x_1h=x_2$, $h^{-1}x_2h=x_3$ and $h^{-1}x_3h=x_1$.

Now we assume that G is a T-group. Then the element of order three of a generating system may be assumed of the form hu ($u\in S_1\times S_2\times S_3$). First we show that there exists an element v in $S_1\times S_2\times S_3$ such that $hu=v^{-1}hv$. Namely let $u=x_1y_2z_3$, $x_1\in S_1$, $y_2\in S_2$ and $z_3\in S_3$. Then since $1=huhuhu=h^2z_1x_2y_3uhu=y_1z_2x_3z_1x_2y_3x_1y_2z_3=y_1z_1x_1z_2x_2y_2x_3y_3z_3$, we have that $y_1z_1x_1=z_2x_2y_2=x_3y_3z_3=1$. So if we let $v=y_2x_3^{-1}$, then $v^{-1}hv=x_3y_2^{-1}hy_2x_3^{-1}=hx_1y_3^{-1}y_2x_3^{-1}=hx_1y_2y_3^{-1}x_3^{-1}=hx_1y_2z_3$ as required. Thus taking $v^{-1}w_iv$ instead of w_i ($w_i\in S_i$; $i=1, 2, 3$), without loss of generality we may assume that $u=1$. Next let the element a of order two which generates G together with h be of the form $a=b_1c_2d_3$, where $b_1\in S_1$, $c_2\in S_2$ and $d_3\in S_3$. Then $h^{-1}ah=d_1b_2c_3$ and $hah^{-1}=c_1d_2b_3$. Since a, $h^{-1}ah$ and hah^{-1} generate $S_1\times S_2\times S_3$, we obtain that $S=\langle b, c, d\rangle$, which implies that the orders of b, c and d equal two. Now let $W(X, Y, Z)$ be any element of a free group on free generators X, Y and Z. Then the mappings $W(b_1, d_1, c_1)\gamma_1=W(c_2, b_2, d_2)$ from S_1 to S_2, $W(c_2, b_2, d_2)\gamma_2=W(d_3, c_3, b_3)$ from S_2 to S_3, and $W(d_3, c_3, b_3)\gamma_3=W(b_1, d_1, c_1)$ from S_3 to S_1 are product preserving. If there exists an automorphism of S permuting b, c and d cyclically, then γ_i is an isomorphism ($i=1, 2, 3$). But then a, $h^{-1}ah$ and hah^{-1} cannot generate $S_1\times S_2\times S_3$.

Conversely if S satisfies (!), then h and $a=b_1c_2d_3$ generate G. In fact, in the above notation, γ_i cannot be an isomorphism, and hence γ_i is not one-to-one ($i=1, 2, 3$). Then since S is simple, the kernel of γ_i is the whole group ($i=1, 2, 3$). So $\langle a, h\rangle$ contains elements of the form b_1x_2 and c_1y_3, where $x_2\in S_2$ and $y_3\in S_3$. Then the commutator of these two elements is a non-trivial element w_1 of S_1. Now taking the conjugates of w_1 successively by a, $h^{-1}ah$ and hah^{-1}, we see that all conjugates of w_1 belong to $\langle a, h\rangle$. Since S_1 is simple, S_1 is contained in $\langle a, h\rangle$. Similarly S_2 and S_3 are contained in $\langle a, h\rangle$.

If there is an automorphism of S permuting b, c and d cyclically, then a, $h^{-1}ah$ and hah^{-1} generate a subgroup of $S_1\times S_2\times S_3$ isomorphic with S. This completes the proof of Proposition 1.

Remark 1. Let a non-Abelian simple group S be generated by three elements b, c and d of order two. Then bc and db generate S.

Proof. Since S is simple, it suffices to show that $\langle bc, db\rangle$ is normalized by b, c and d. Now bc and db are inverted by b and c, and by d and b respectively. Since $dbbc=dc$, we have that $cdbc=(dbbc)^{-1}bc$ and $abcd=db(dbbc)^{-1}$. So c and d normalize $\langle bc, db\rangle$.

From now on we assume that S is a non-Abelian simple group satisfying the condition (!), and that G is the wreath product of S with respect to a cyclic group $H=\langle h\rangle$ of order three. Then, for simplicity, we call G a *TW-group* and the T-graph Γ corresponding to G a *TW-graph.*

Proposition 2. *A TW-graph Γ is not bipartite.*

Proof. Assume that Γ is bipartite. Let $E(H)$ be the set of vertices Hx such that the distance $\partial(H, Hx)$ is even and $O(H)$ the set of vertices Hy such that $\partial(H, Hy)$ is odd. Now if z is an element of G such that $E(H)z\cap E(H)\neq\emptyset$, then $E(H)z=E(H)$. In fact, otherwise, there exist vertices Hu and Hv such that $\partial(H, Hu)=\partial(H, Hv)=\partial(Hz, Hu)\equiv 0 \pmod 2$ and $\partial(Hz, Hv)\equiv 1 \pmod 2$. So we have a closed walk of odd length initiating and terminating at H and passing through Hu, Hz and Hv. But a closed walk of odd length contains a circuit of odd length. This contradicts the fact that Γ is bipartite ([1], p. 50). Since $\partial(H, Hx)=\partial(Ha, Hxa)$ and $\partial(H, Ha)=1$, $E(H)a=O(H)$. So $E(H)$ is a set of imprimitivity for G of length two. Thus G contains a normal subgroup of index two. This is a contradiction.

The girth and diameter of Γ are intimately related with the word problem in G with a and h alphabets. Any word $W(a, h)$ which is not equal to 1, a^2 or h^3 such that $HW(a, h)=H$ corresponds to a closed walk in Γ. Clearly we may assume that $W(a, h)=ah^{e(1)}\cdots ah^{e(r)}$, where $e(i)=1$ or 2 $(1\leqq i\leqq r)$. Then the length of the corresponding closed walk is equal to r. Let g be the girth of Γ. Then we have the following inequality

$$g\leqq |ah|.$$

For TW-graphs we might have the equality here. But this is not always true for general T-graphs.

Example. Let p be an odd prime such that $p \equiv 2 \pmod 3$. Then the following relations define a group G^* of order $6p^2$:

$$b^2 = c^3 = 1,\ bcb = c^{-1},\ d_1^p = d_2^p = 1,\ d_1 d_2 = d_2 d_1,\ bd_1 b = d_2,$$
$$bd_2 b = d_1,\ c^{-1} d_1 c = d_2, \text{ and } c^{-1} d_2 c = d_1^{-1} d_2^{-1}.$$

G^* is a T-group. In fact, let $a = d_1 d_2^{-1} b$ and $h = c$. Then $a^2 = 1$ and $aha = d_1 d_2^{-1} bcd_1 d_2^{-1} = d_1 d_2^{-1} bcd_2 d_1^{-1} = d_1 d_2^{-1} c^{-1} d_2 d_1^{-1} = c^{-1} d_1^{-1} d_2^{-1} d_1^{-1} d_2 d_1^{-1} = c^{-1} d_1^{-3}$. Thus $\langle a, h \rangle$ contains d_1 and hence d_2. Furthermore, $ahah = d_1 d_2^{-1} bcd_1 d_2^{-1} bc = d_1 d_2^{-1} c^{-1} d_2 d_1^{-1} = d_1 d_2^{-1} d_1^{-1} d_2^{-1} d_2^{-1} = d_2^{-3}$. Thus the order of ah is equal to $2p$. On the other hand, $ahah^2 = d_1 d_2^{-1} bcd_1 d_2^{-1} bc^{-1} = d_1 d_2^{-1} c^{-1} d_2 d_1^{-1} c^{-1} = d_1 d_2^{-1} d_1^{-1} d_2^{-1} d_2^{-1} c = d_2^{-3} c$ and $d_2^{-3} cd_2^{-3} cd_2^{-3} c = d_2^{-3} d_1^{-3} d_1^{3} d_2^{3} = 1$. Thus we have that $g \leqq 6$.

Proposition 3. *If Γ is a TW-graph, then $g \geqq 6$.*

Proof. If $g = 3$, then we have a relation in G of the form $ah^{e_1} ah^{e_2} ah^{e_3} = 1$, where $e_i = 1$ or 2 $(i = 1, 2, 3)$. Since a belongs to $S_1 \times S_2 \times S_3$, we have that $e_1 + e_2 + e_3 \equiv 0 \pmod 3$. So we have that either $e_1 = e_2 = e_3 = 1$ or $e_1 = e_2 = e_3 = 2$. In the first case, $aha = h^{-1} ah^{-1}$. So, $hah^{-1} ahah^{-1} a = hah^{-1} h^{-1} ah^{-1} h^{-1} a = 1$. Thus hah^{-1} and a commute. Conjugating by h and h^{-1}, we see that $\langle a, hah^{-1}, h^{-1} ah \rangle$ is an Abelian normal subgroup of G. This is a contradiction. Taking h^{-1} instead of h, the second case can be similarly treated as the first case.

If $g = 4$, then we have a relation in G of the form $ah^{e_1} ah^{e_2} ah^{e_3} ah^{e_4} = 1$, where $e_i = 1$ or 2 $(i = 1, 2, 3, 4)$. Since a belongs to $S_1 \times S_2 \times S_3$, we have that $e_1 + e_2 + e_3 + e_4 \equiv 0 \pmod 3$. So we may assume that either $ahah^{-1} ahah^{-1} = 1$ or $ahah^{-1} ah^{-1} ah = 1$. In the first case, a and hah^{-1} commute. So we get the similar contradiction as in the case of $g = 3$. In the second case, $aha = h^{-1} ahah$. So $\langle aha, h \rangle$ is an Abelian normal subgroup of G. This is a contradiction.

If $g = 5$, then we have a relation in G of the form $ah^{e_1} ah^{e_2} ah^{e_3} ah^{e_4} ah^{e_5} = 1$, where $e_i = 1$ or 2 $(i = 1, 2, 3, 4, 5)$. Since a belongs to $S_1 \times S_2 \times S_3$, we have that $e_1 + e_2 + e_3 + e_4 + e_5 \equiv 0 \pmod 3$. So we may assume that either $ahah^2 ah^2 ah^2 ah^2 = 1$ or $ahahahahah^2 = 1$. In the first case, $\langle ah^2 \rangle$ contains ah and hence h. This is a contradiction. The second case is similar to the first case.

Proposition 4. *For $b \in S_1 \times S_2 \times S_3$ $Hbh = Hb$ if and only if b commutes with h. If $b \neq 1$ commutes with h, then we have a circuit of even length with H and Hb as initial and terminal vertices.*

Proof. $Hbh=Hb$ if and only if $bhb^{-1}\in H$. Since $bhb^{-1}=bhb^{-1}h^{-1}h$ ahd $bhb^{-1}h^{-1}\in S_1\times S_2\times S_3$, $Hbh=Hb$ if and only if $bhb^{-1}h^{-1}=1$.

Assume that $\partial(Hb, Hx)=1$, where $x\in S_1\times S_2\times S_3$. If $\partial(H, Hx)=\partial(H, Hb)=1$, then $Hxh=Hx$. So x commutes with h. Since $Hx=Hab$, $Hahb$ or Hah^2b, this implies that a commutes with h. This is a contradiction.

Remark 2. It is difficult for the author to evaluate $\partial(H, Hb)$.

3. Automorphism groups of *TW*-graphs.

Proposition 5. *Let G^* be a T-group and Γ the corresponding T-graph. Let $\mathfrak{G}$ be the automorphism group of Γ ahd $\mathfrak{H}$ the stabilizer of the vertex H in $\mathfrak{G}$. Then the order of $\mathfrak{H}$ equals 2^m3, where m is a non-negative integer, and so $\langle h\rangle$ is a Sylow 3-subgroup of $\mathfrak{H}$.*

Proof. If σ is an element of $\mathfrak{H}$, then $H\sigma=H$ and $(Ha)\sigma\in\{Ha, Hah, Hah^2\}$. Therefore if the order of σ equals a prime larger than three, σ leaves Ha, Hah and Hah^2 invariant. So, using the recurrence argument with respect to the distance from H, we obtain that $\sigma=1$, a contradiction.

Now let σ be an element of a Sylow 3-subgroup of $\mathfrak{H}$ containing h. Since we can replace σ by σh or σh^2, if need be, we may assume that $(Ha)\sigma=Ha$. Then we have that $(Hah)\sigma=Hah$ and $(Hah^2)\sigma=Hah^2$. So, as above, we obtain that $\sigma=1$.

Proposition 6. *Let σ be an automorphism of a T-group G^*. Then σ is an automorphism of the corresponding T-graph Γ if and only if $H\sigma=H$ and $\{Ha, Hah, Hah^2\}\sigma=\{Ha, Hah, Hah^2\}$.*

Proof. If σ satisfies the stated condition, then, for any $b\in G^*$, $(Hb)\sigma=H(b\sigma)$ and $(Hah^ib)\sigma=Hah^j(b\sigma)$ $(i, j=0, 1, 2)$. So σ preserves the adjacency of Γ.

If σ is an automorphism of Γ, then $H\sigma=Hb$ for some $b\in G^*$. Since $H\sigma$ is a subgroup of G^*, $H\sigma=H$. Hence $\{Ha, Hah, Hah^2\}\sigma=\{Ha, Hah, Hah^2\}$.

Now let G be a TW-group, Γ the corresponding TW-graph and $\mathfrak{G}$ the automorphism group of Γ. Then, by Proposition 5, $[\mathfrak{G}:G]=2^e$. It seems, however, not easy to give an effective bound on e. Our proof to the following proposition is based on the classification of non-Abelian simple groups of finite orders.

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Proposition 7. *In the notation above,* $e \leqq 1$. $e=1$ *if and only if, in the notation of Proposition* 1, *there exists an automorphism* τ *of* S *such that* τ *fixes one of three generating elements* b, c *and* d *of* S *of order two and interchanges the remaining two.*

Remark 3. If τ fixes b and interchanges c and d, then $|bc|=|bd|$. Therefore, if $|bc|$, $|cd|$ and $|db|$ are distinct, then $e=0$, namely $\mathfrak{G}=G$.

Proof of Proposition 7. The main point of the proof is to show that $\mathfrak{G}$ is not simple. First we notice that for any prime p such that (1) p divides the order of $\mathfrak{G}$ and (2) p is larger than three a Sylow p-subgroup of $\mathfrak{G}$ is, by Proposition 5, isomorphic to a direct product of three Sylow p-subgroups of S.

Now we assume that $\mathfrak{G}$ is simple.

$\mathfrak{G}$ is not isomorphic to any one of twenty-six sporadic simple groups. In fact, for any sporadic simple group the largest prime divisor of the order occurs only to the first power.

$\mathfrak{G}$ is not isomorphic to any one of alternating groups of degree $n \geqq 5$. In fact, by Bertrand's postulate ([7], p. 189) there exists a prime $p \geqq 5$ such that $n \geqq p$ and $2p > n$. Then a Sylow p-subgroup is cyclic of order p.

Hence $\mathfrak{G}$ must be isomorphic to one of simple groups of Lie type.

(i) $\mathfrak{G}$ is not isomorphic to $A_l(q)$. In fact, if $l=1$, then all Sylow subgroups whose orders are odd and prime to q are cyclic. If only Sylow 3-subgroups appear among them, then we have that either i) $\frac{1}{2}(q+1)=2^m$ and $\frac{1}{2}(q-1)=3^n$ or ii) $\frac{1}{2}(q+1)=3^m$ and $\frac{1}{2}(q-1)=2^n$. Easily we get $q=7$ for case i) and $q=5$ or 17 for case ii). Then a Sylow q-subgroup is cyclic. So we may assume that $l>1$. Now $A_l(q)$ contains a cyclic subgroup of order $(q^{l+1}-1)/(q-1)d$, where d is the greatest common divisor of $l+1$ and $q-1$. Then by a theorem of K. Zsigmondy [10], unless $q=2$ and $l=5$, there exists a prime divisor $s \geqq 5$ of $(q^{l+1}-1)/(q-1)d$ such that the order of q with respect to s equals $l+1$. Then a Sylow s-subgroup is cyclic. If $q=2$ and $l=5$, then a Sylow 31-subgroup is of order 31.

(ii) $\mathfrak{G}$ is not isomorphic to ${}^2A_l(q)$, $l \geqq 2$. In fact, ${}^2A_l(q)$ contains a cyclic subgroup of order $(q^{l+1}+(-1)^l)/(q+1)d$, where d is the greatest common divisor of $l+1$ and $q+1$ ([8], p. 190). So we can argue as in (i).

(iii) $\mathfrak{G}$ is not isomorphic to $B_l(q)$, $l \geqq 2$. In fact, $B_l(q)$ involves $A_{l-1}(q)$. Unless $q=2$ and $l=6$, by (i), $A_{l-1}(q)$ contains a cyclic Sylow s-subgroup S where the order of q with respect to s equals l. Then a Sylow

s-subgroup of $B_l(q)$ has order $|S|^2$. If $q=2$ and $l=6$, then a Sylow 31-subgroup js of order 31.

(iv) $\mathfrak{G}$ is not isomorphic to ${}^2B_2(2^{2m+1})$, since all Sylow subgroups of odd orders of ${}^2B_2(2^{2m+1})$ are cyclic.

Now the remaining cases can be treated as in (iii). Hence we only list involvolvement pairs. For this see [2] and [4]. (v) $C_l(q)$ involves $A_{l-1}(q)$, $l \geqq 3$. (vi) $D_l(q)$ involves $A_{l-1}(q)$, $l \geqq 4$. (vii) ${}^2D_l(q)$ involves $B_{l-1}(q)$, $l \geqq 4$. (viii) ${}^3D_4(q)$ involves $G_2(q)$. (ix) $E_6(q)$ involves $A_5(q)$. (x) $E_7(q)$ involves $A_7(q)$. (xi) $E_8(q)$ involves $A_8(q)$. (xii) ${}^2E_6(q)$ involves $B_4(q)$. (xiii) $F_4(q)$ involves $B_4(q)$. (xiv) ${}^2F_4(q)'$ involves ${}^2B_2(q)$. (xv) $G_2(q)$ involves $A_2(q)$. (xvi) ${}^2G_2(q)$ involves $A_1(q)$.

Thus we have a contradiction. Therefore $\mathfrak{G}$ is not simple. Let N be a maximal normal subgroup of $\mathfrak{G}$. If $N \cap (S_1 \times S_2 \times S_3)=1$, then $\mathfrak{G}/N$ is simple and $\mathfrak{G}/N$ contains a subgroup $N(S_1 \times S_2 \times S_3)/N$ isomorphic to $S_1 \times S_2 \times S_3$. So by repeating the above argument we get a contradiction. Therefore we have that $N \cap (S_1 \times S_2 \times S_3) \neq 1$. Since S is non-Abelian simple, it implies that N contains $S_1 \times S_2 \times S_3$. If $N \neq S_1 \times S_2 \times S_3$, then we can apply the same argument to N instead of $\mathfrak{G}$. Thus we obtain that $S_1 \times S_2 \times S_3$ is normal in $\mathfrak{G}$.

$\mathfrak{H}$ acts on $\{Ha, Hah, Hah^2\}$. The stabilizer K of Ha in $\mathfrak{H}$ is a Sylow 2-subgroup of $\mathfrak{H}$. K acts on $\{Hah, Hah^2\}$. Let L be the kernel of this representation. Then the index of L in K equals 1 or 2, and L is normal in $\mathfrak{H}$. We consider the subgroup $HL(S_1 \times S_2 \times S_3)$. Since $HaLa=Haa=H$, we have that $aLa=L$. Since $h^{-1}Lh=L$ and $G=\langle a, h\rangle$, L is normal in $\mathfrak{G}$. It implies that $L=1$. This completes the proof of the first half of Proposition 7.

Now we assume that $e=1$. Let t be an element of order two of $\mathfrak{H}$. Then by the argument in the last paragraph we have that $tht=h^2$. By Proposition 6 we may assume that $Hat=Ha$. Then we have that $ata=t$, $h^{-1}th$ or hth^{-1}. If $ata=h^{-1}th=ht$, then $t=ahaht$, which implies that $ahah=1$. This is a contradiction. Similarly we can show that $ata \neq hth^{-1}$. Thus we obtain that $ata=t$. Since $tht=h^2$, we may assume that t leaves S_1 invariant and interchanges S_2 and S_3. Recalling $a=b_1c_2d_3$ we have that $tb_1t=b_1$, $tc_2t=d_3$ and $td_3t=c_2$. Moreover, $th^{-1}aht=td_1b_2c_3t=htath^{-1}=hah^{-1}=c_1d_2b_3$. Thus we obtain that $td_1t=c_1$, $tc_3t=d_2$ and $tb_2t=b_3$. Therefore S possesses an automorphism fixing b and interchanging c and d.

Conversely we assume that S possesses an automorphism fixing b and interchanging c and d. Then the relations $tb_1t=b_1$, $tc_1t=d_1$, $td_1t=c_1$, $tb_2t=b_3$, $tc_2t=d_3$, $td_2t=c_3$, $tb_3t=b_2$, $tc_3t=d_2$ and $td_3t=c_2$ define an automorphism of $S_1 \times S_2 \times S_3$. We can check easily that $tht = h^{-1}$ and $tat = a$. Therefore, by Proposition 6, t is an automorphism of the TW-graph Γ.

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ERRATA

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Page 145, line 21. For "(!) ...b, c and d such that..." read "(!) ...b, c and d of order two such that...".